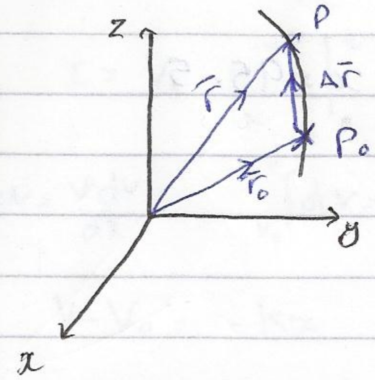
Alaa Eldin Tamer

Sec 13

Curvilinear motion



$$\bar{D} = \bar{\Delta r} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}$$

$$\bar{D} = \bar{\Delta r} = \bar{r} - \bar{r}_0$$

$\bar{\Delta r} = \bar{D} =$ displacement

$$\langle \bar{v} \rangle = \lim_{\Delta t \to 0} \frac{\bar{\Delta r}}{\Delta t} \Rightarrow \bar{V} = \frac{d\bar{x}}{dt}$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z} k$$

$$= V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

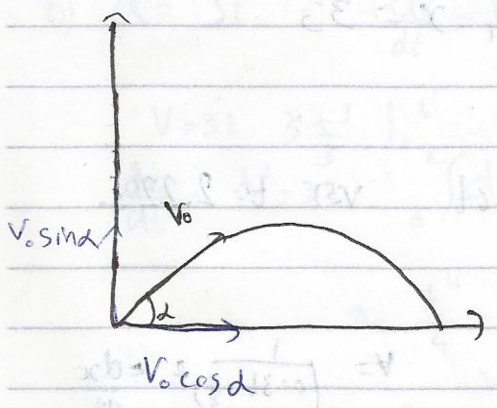
$$\boxed{\begin{aligned} \dot{x} &= \frac{dx}{dt} \\ \dot{y} &= \frac{dy}{dt} \\ \dot{z} &= \frac{dz}{dt} \end{aligned}}$$

$$V = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$$

projectiles



$$a = \ddot{x}\hat{i} + \ddot{y}\hat{j} \qquad \ddot{x} = 0 \qquad \ddot{y} = -g$$

$$\frac{d\dot{x}}{dt} = 0 \qquad \dot{x} = \text{constant}$$

$$\boxed{\dot{x} = V_0 \cos\alpha}$$

$$\frac{dx}{dt} = V_0 \cos\alpha \qquad \int dx = V_0 \cos\alpha \int dt$$

$$\boxed{x = V_0 \cos\alpha \, t}$$

$$\ddot{y} = -g = \frac{d\dot{y}}{dt} \qquad \int d\dot{y} = -g \int dt \qquad \dot{y} = -gt + C$$

$$V_0 \sin\alpha = 0(t) + C$$

$$C = V_0 \sin\alpha \Rightarrow \boxed{\dot{y} = -gt + V_0 \sin\alpha}$$

$\dot{y} = \frac{dy}{dt} = V_o \sin\alpha - gt$ $\quad \int dy = \int V_o \sin\alpha - gt \, dt$

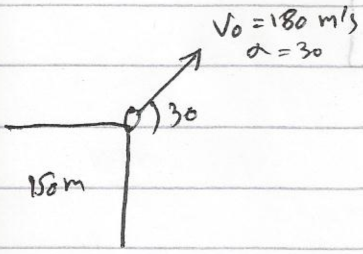
$y = V_o \sin\alpha \, t - \frac{gt^2}{2} + C$ $\quad$ but $C = 0$

(initial position is origin)

$$\boxed{y = V_o \sin\alpha \, t - \frac{gt^2}{2}}$$

Ex 11.7

a) find distance to position where particle hits the ground



$y = V_o \sin\alpha \, t - \frac{gt^2}{2}$

$-150 = 180 \sin 30 \, t - \frac{9.81}{2} t^2$

$t = 19.9$

$X = V_o \cos\alpha \, t = 180 \cos 30 \times 19.9 = 3102 \text{ m}$

b) find velocity when it hits the ground

$\dot{X} = V_o \cos\alpha = 180 \cos 30 = 90\sqrt{3}$

$\dot{Y} = V_o \sin\alpha - gt = 180 \sin 30 - (9.81 \times 19.9) = -105 \text{ m/s}$

$V = \sqrt{(90\sqrt{3})^2 + (105)^2} = 188 \text{ m/s}$

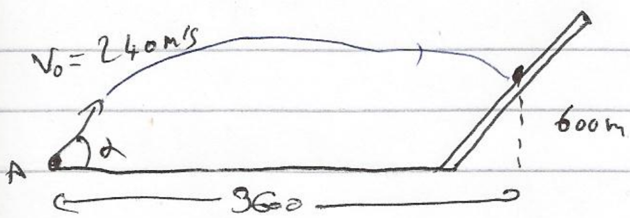
c) find highest distance above ground

$\dot{y} = V_o \sin\alpha - gt = 0 \quad t = 9.17$

$Y = -\frac{gt^2}{2} + V_o \sin\alpha \, t$

$Y = \frac{-9.81(9.17)^2}{2} + 180 \sin 30 \times 9.17 = 412.5$

Ex 11.8



find α

$X = V_o \cos\alpha \, t \quad 3600 = 240 \cos\alpha \, t$

$t = \frac{15}{\cos\alpha}$

$Y = -\frac{gt^2}{2} + V_o \sin\alpha \, t$

$600 = -4.95 \left(\frac{15}{\cos\alpha}\right)^2 + 240 \times 15 \tan\alpha$

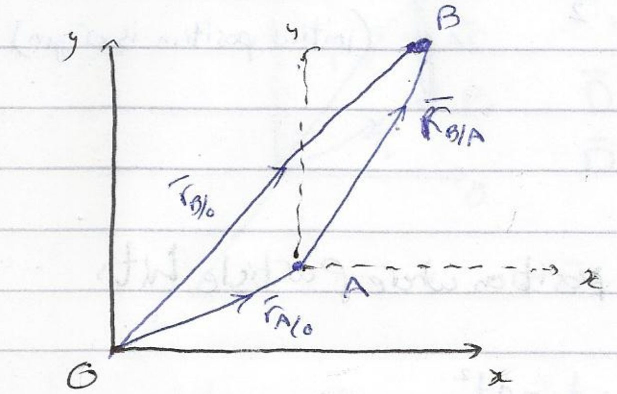
$600 = -1104 (1 + \tan^2\alpha) + 3600 \tan\alpha$

$-1104 \tan^2\alpha + 3600 \tan\alpha - 1704 = 0$

$\tan\alpha = 2.69, \; 0.57$

$\alpha = 69.6, \; 29.7$

Relative motion



$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

$$\boxed{\begin{aligned} \bar{r}_{B/A} &= \bar{r}_B - \bar{r}_A \\ \bar{V}_{B/A} &= \bar{V}_B - \bar{V}_A \\ \bar{a}_{B/A} &= \bar{a}_B - \bar{a}_A \end{aligned}}$$

Ex 11.9 $V_A = 10 m/s$ $\qquad r_A = 10 \times 5 = 50 \hat{i}$

$V_B = at + u = 1.2 \times 5 = 6 m/s$ $\qquad V_B = -6 j$

$y_B = y_0 - \frac{1}{2} a t^2 = 35 - \frac{1}{2} \times 1.2 \times 5^2 = 20 m$

$r_B = -20 \hat{j}$ $\qquad a_B = -1.2 \hat{j}$

$r_{B/A} = -20 \hat{j} - 50 \hat{i}$

$V_{B/A} = -6 \hat{j} - 10 \hat{i}$

$a_{B/A} = -1.2 \hat{j}$